ARTICLE IV.

RESULTS OF RECENT RESEARCHES ON THE EVOLUTION OF THE STELLAR SYSTEMS.

(Plates IV and V.)

BY T. J. J. SEE, A.M., PH.D. (BERLIN),
ASTRONOMER AT THE LOWELL OBSERVATORY.

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It is now two hundred and eleven years since Newton published the *Principia*, embodying his grand generalization of the law of gravitation, and the proof of this law for the most obvious and fundamental phenomena of the solar system. Geometers have since been occupied with the development and extension of the principle discovered by the illustrious Newton, and have finally explained with almost entire satisfaction the motions and attractions of the planets, satellites, comets, and other bodies which revolve about the sun. This great development can hardly fail to excite the admiration of those who contemplate the history of scientific progress, and must be accounted one of the most noble and enduring monuments of the human mind. So sublime an achievement has required the combined labors of a long series of men of transcendent mathematical and mechanical genius, each building upon the foundation laid by his predecessors. Though many distinguished geometers have borne an honorable part in this remarkable development of Physical Astronomy, it will not be inappropriate to point out the great credit for the perfection of the Newtonian theory due to Clairaut and Euler, Lagrange and Laplace, Gauss and Hansen, Adams and Leverrier. Among living investigators in mathematical astronomy the names of Hill and Newcomb, Darwin and Poincaré occupy the foremost place. These great men have brought the mechanics of the heavens to so high a state of perfection that in almost every case we may now predict the heavenly motions as accurately as we can observe them. In view of the rapid perfection of telescopes and other instruments of precision, this achievement, from the intricacy of the analysis required in the problem, and the abstruseness of the methods used in the reduction of observations, must be ranked as incomparably the most profound yet attained in any branch of Physical Science.

Notwithstanding these splendid triumphs of the science of Celestial Mechanics, an even greater and more recondite work remains to be done in a closely related field. This is the investigation of the origin and cosmical history of the planetary and other systems observed in the immensity of space. Even if some credit for pioneer work on this problem be assigned to Kant, or, more remote still, to the Greeks of the pre-Socratic age, it yet remains true that Laplace is the real discoverer to whom we are indebted for the first ideas which proved fruitful for the advancement of science. About a century ago this great geometer outlined for the solar system the celebrated Nebular Hypothesis, upon which nearly all subsequent investigation has been based, and which has since been substantially confirmed, though but very little modified until within the last twenty-five years. Passing over as irrelative in the present discussion the early work of Herschel and Rosse, Helmholtz and Kelvin, Newcomb and Lane, we come down to the modifications introduced by Darwin about 1880.

In establishing the theory of gravitation, Newton assigned also the true cause of the tides of the seas, though his explanation carried with it all the defects of the equilibrium theory. More than a century passed before the dynamical character of the problem of the oceanic tidal oscillations was clearly perceived, when Laplace developed and applied the true theory with all the penetration characteristic of that great mathematician. Yet in spite of the profundity which marks his treatment of the tides of the oceans, it seems never to have occurred to him, or at least he made no record of the fact, that the attraction of the moon necessarily produces tides in the body of, as well as in the aqueous layers covering, the earth. We need not be surprised at this omission on the part of Laplace and those who followed him, if we recall that for many years after the perfection of Analytical Mechanics by D'Alembert and Lagrange, the subject was treated wholly from the point of view of material particles, and the resulting system was what is now called Rigid Dynamics. Little attention was bestowed upon the theory of fluid motion, partly because of its intricacy, and partly because there were no obvious applications of the results except in the case of the tides, already treated by Laplace with great penetration and extreme generality. As mathematicians since the time of Newton had been occupied chiefly with the development of the theory of planetary perturbations along the line of rigid dynamics, it did not occur to them that they were building on a false premise, that in reality the heavenly bodies so far as known are not solid, but fluid, though Laplace with his usual sagacity had long foreseen that in the case of our planets the nuclei are covered with fluid layers held in equilibrium by the pressure and attraction of their parts. His grand treatment in the *Mécanique Céleste* recognizes the fluidity of the envelopes of the planets, and exhaustively examines the oscillations that will arise therein. Nor did he fail to consider fully the deviations from spherical form and the probable laws of density for the layers which compose the bodies of the planets.

The effect of so monumental a work as the *Mécanique Céleste* was twofold: on the one hand it brought Physical Astronomy to an unexpected state of perfection, while on the other it produced the impression on the less creative minds that there were no great problems untouched by the master-mind of Laplace. His work had indeed well-nigh exhausted the theory of Celestial Mechanics, so far as it could be built upon the assumptions of rigid dynamics; at least subsequent work has been for the most part little more than refinement or perfection of the methods and processes given in the *Mécanique Céleste.* The work of Laplace was designed for the solar system, and the idea that the universe is really composed of fluid bodies, self-luminous stars and nebulæ in space, seems never to have occurred to him, or he would have foreseen that however adequate Rigid Dynamics may be for effecting a first approximation, the true theories of ultimate Celestial Mechanics must be founded upon the laws of viscous fluids in motion. So great is the influence of tradition that it is difficult for us to realize fully that the stars and nebulæ are viscous fluids, self-luminous liquid or gaseous masses, and that even in the solar system the bodies are all fluids of various viscosities. This new point of view respecting the actual facts of the universe has brought about an important modification in the nebular hypothesis and in the ultimate theories of Celestial Mechanics, of which we shall now give some account.

About 1875, G. H. Darwin, who had qualified himself for the Law and been called to the Bar, on account of ill-health, abandoned his profession to undertake for Lord Kelvin some scientific work, which among other things included the reduction of a great mass of Indian tide observations with a view of throwing light upon the problem of the rigidity of the earth. This work, besides leading Lord Kelvin to the celebrated conclusion that the earth as a whole is "probably more rigid than steel, but not quite so rigid as glass," was the occasion* of the younger Darwin developing the theory of bodily tides, or the theory of the tides which would arise in the earth on supposition that it is not rigid as at present, but a viscous fluid, as it must have been, according to Laplace, at some past age. While some allusions to bodily tides can be found in scientific literature as far back as Kant, and especially in the papers of Delaunay on the secular acceleration of the moon's

* In the *Atlantic Monthly*, for April, 1898, Prof. Darwin remarks: "It was very natural that Mr. See should find in certain tidal investigations which I undertook for Lord Kelvin the source of my papers, but as a fact the subject was brought before me in a somewhat different manner. Some unpublished experiments on the viscosity of pitch induced me to extend Lord Kelvin's beautiful investigation of the strain of an elastic sphere to the tidal distortion of a viscous planet. This naturally led to the consideration of the tides of an ocean lying on such a planet, which forms the subject of certain paragraphs now incorporated in Thomson and Tait's *Natural Philosophy*.

mean motion, it is yet indisputable that Darwin was the first writer to treat the problem in a systematic, thorough-going and original way. Recognizing that at some epoch in the past, the earth was probably a mass of viscous fluid, he set for himself this problem: To determine the bodily tidal distortion of the earth, and the effects of this alteration of figure upon the orbital motion of the moon, and upon the earth's rotation. His papers were communicated to the Royal Society between 1878 and 1882, and are celebrated contributions to the general theory of tides. In these papers he has traced the moon back to close proximity to the earth, when the two, at the breaking off of the moon, were most probably revolving in about 2 h. 41 m. The moon has since receded from the earth under the action of tidal friction, while the rotation of the earth has been slowed up in corresponding degree. It was rendered certain that in the origin of the Lunar-Terrestrial System, the action of tidal friction had played a prominent, if not a paramount part, and the question naturally arose whether it had not been equally potent in the development of other parts of the solar system. When, however, Prof. Darwin came to apply the results to other satellite systems and to the solar system as a whole, it was found that here the effects had been much less considerable than in the case of the earth and moon, owing chiefly to the small masses of the attendant bodies. Thus the major axes of the orbits had perhaps been very slightly increased, and the rotations correspondingly exhausted, but no radical change had taken place. Under these circumstances it was natural that Darwin should drop the subject without further search for extension of the principle he had developed.

About November 1, 1888, while I was still an undergraduate at the Missouri State University I became much interested in the origin of the double stars. The immediate cause of my taking up the subject was the Missouri Astronomical Medal, occasionally awarded by the University to a graduate of highest standing in the Mathematical and Physical Sciences. Having been informed by Prof. W. B. Smith that I was eligible to write for the medal, by virtue of my standing in the Physical Sciences, our conversation drifted on to the probable subject of the Thesis, and in this way he was led to suggest a *criticism* of Darwin's work on the origin of the moon. He remarked: "You may find this only a pocket, already worked out, and not a continuous vein of rich ore, but it seems to me worth thinking of. At any rate I would not advise you to write on the orthodox Laplacean Nebular Hypothesis, for that subject is worn threadbare."

The suggestion of a *critique* of Darwin's work did not quite meet my approval, for I feared the subject was already exhausted and would leave no field for future progress. As I had been observing various double stars for the past two years, and had seen no suggestion regarding their mode of development, it occurred to me that perhaps the tidal theory might find application among the stars. When I had collected such orbits as were available in the books at my disposal (Humboldt's *Cosmos*, Herschel's *Outlines*, etc.), I discovered to my surprise that unlike the orbits of the planets and satellites, they are very eccentric, though not so eccentric as those of the periodic comets. It was at once evident that it would be hopeless to attempt to explain the origin of the stellar systems, if we could not explain the cause of the high eccentricities of the orbits. The next day I called on Prof. Smith and told him of the discovery that the orbits are very eccentric, and asked whether he thought I might explain this peculiarity on the tidal theory; rubbing his head for a moment in quiet reflection, he replied: "Oh! I see what you mean; you think the dragging of the tides in the bodies of the stars has produced the elongation you find in the orbits. Such an idea can hardly be discussed off-hand, but it is at least worth examining; it may prove fruitful." "That is exactly what I mean," said I, "and you have correctly interpreted my line of thought." After this conversation, which is here reported exactly as it occurred,* there was nothing else before my mind for several days, as I was wholly occupied with finding out whether the problem undertaken was soluble, and, if so, whether it would result in any important Physical Truth. Having established the fact of high eccentricity as thoroughly as the published orbits at my disposal would admit, I set about that same day the problem of explaining the cause of the eccentricities; and as I worked the impression continued to grow on the mind that since the stars are not solid, but self-luminous fluid bodies like our sun, and the two members of a system comparable in mass, the action of each body would produce tides in the other, and the lagging of the tides in the two stars would gradually expand and elongate the orbits as now observed in space. And before I had obtained access to the learned papers which Darwin had communicated to the Royal Society, or even to his article "Tides" in the *Encyclopædia Britannica,* I proved by an elementary process that when the bodies rotate more rapidly than they revolve, the eccentricity of the orbit would gradually increase. Here then was a result confirmatory of the happy intuition, and for the past nine years my energies have been largely devoted to the extension and generalization of the theory of bodily tides in relation to cosmical evolution.

After concluding my undergraduate studies at the University of Missouri, I continued the work at the University of Berlin. It is particularly of that work and the extension which I have since made of it that I shall speak to-night. The theory of tidal friction developed in the *Inaugural Dissertation* presented to the Faculty of the University of Berlin is essentially a special treatment of the general theory as it occurs in nature, while that previously developed by Darwin in connection with the moon and planets is restricted by the condition that the perturbing body is very small. I shall therefore discuss the general case as presented in my own researches.

*As the occasion of my beginning this work has never been published, I trust it will not be thought inappropriate for me to recall it in this paper to the American Philosophical Society.

Suppose we denote an element of the mass of a spheroid by m, and its distance from the axis of rotation by d; then the moment of inertia is

$$I = \Sigma md^2$$

If the spheroid be rotating with an angular velocity y, then Iy will be the moment of momentum of the body about its axis. For a second body whose moment of inertia is I', and angular velocity z, the moment of momentum is $I'z$.

Following the analogy of Darwin's procedure, we choose a system of units designed to simplify the resulting equations. Let us take as the unit of mass

$$\frac{M\,M'}{M+M'};$$

and as the unit of length a space Γ such that the moment of inertia of the spheroid about its axis of rotation shall be equal to the moment of inertia of the two spheroids treated as material points, about their common centre of inertia when distant apart Γ. Then we have

$$M\left\{\frac{M'\Gamma}{M+M'}\right\}^2 + M'\left\{\frac{M\Gamma}{M+M'}\right\}^2 = I, \text{ or}$$

$$\Gamma = \left\{\frac{I\,(M+M')}{MM'}\right\}^{\frac{1}{2}}.$$

Let the unit of time be the interval in which one spheroid describes 57°.3 in its orbital motion about the other when distant Γ. In this case, $\frac{1}{\theta}$ is the orbital angular velocity of the body. The generalization of Kepler's law gives

$$\theta^{-2}\,\Gamma^3 = \mu\,(M + M'), \text{ and}$$

$$\theta = \left\{\frac{I^3\,(M+M')}{\mu^2\,(MM')^3}\right\}^{\frac{1}{4}}$$

Now suppose the two stars to revolve about their common centre of inertia in a circular orbit, with an angular velocity Ω, when the radius vector is ρ. Then the orbital moment of momentum is

$$M\left(\frac{M'\rho}{M+M'}\right)^2\Omega + M'\left(\frac{M\rho}{M+M'}\right)^2\Omega = \left(\frac{MM'}{M+M'}\right)\rho^2\,\Omega.$$

In a circular orbit the law of Kepler gives $\Omega^2\rho^3 = \mu\,(M + M')$; and $\Omega\rho^2 = \mu^{\frac{1}{2}}\,(M + M')^{\frac{1}{2}}\,\rho^{\frac{1}{2}}$; and on inserting for $\Omega\rho^2$ its value, we have $\mu^{\frac{1}{2}}\,MM'\,(M + M')^{-\frac{1}{2}}\,\rho^{\frac{1}{2}}$, which in special units is $\rho^{\frac{1}{2}}$. Now the total moment of momentum of the system is constant, and is given by

$$H = Iy + I'z + \mu^{\frac{1}{2}} MM' (M + M')^{-\frac{1}{2}} \rho^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The kinetic energy of orbital motion is

$$\frac{1}{2} M \left(\frac{M'\rho}{M + M'}\right)^2 \Omega^2 + \frac{1}{2} M' \left(\frac{M\rho}{M + M'}\right)^2 \Omega^2 = \frac{1}{2} \left(\frac{MM'}{M + M'}\right) \rho^2 \Omega^2 = \frac{1}{2} \mu \frac{MM'}{\rho}.$$

The kinetic energy of rotation is

$$\frac{1}{2} I y^2 ; \frac{1}{2} I' z^2.$$

The potential energy of the system is

$$- \mu \frac{MM'}{\rho}.$$

By adding all these energies together we get the total energy of the system :

$$\frac{E}{2} = \frac{1}{2} Iy^2 + \frac{1}{2} I'z^2 - \frac{\mu}{2} \frac{MM'}{\rho},$$

where E is twice the whole energy.

In the system of special units, I, $\mu MM'$, are equal to unity. If we put $k = \frac{I'}{I}$, we shall get

$$E = y^2 + k z^2 - \frac{1}{\rho}.$$

Let $x = \Omega^{-\frac{1}{2}}$, and then $\Omega^{-\frac{1}{3}} = \rho^{\frac{1}{2}}$, $x = \rho^{\frac{1}{2}}$, and we have finally

$$E = y^{\frac{1}{3}} + z^2 - \frac{1}{x^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If we suppose the two stars to turn on their axes in the same time in which they revolve in their orbits, so that they show always one face to each other, the motion of the system will be as if the masses were rigidly connected. This condition is given by

$$\Omega = y = z, \text{ or}$$

$$\Omega^{-\frac{1}{3}} = x = y^{-3} = z^{-\frac{1}{3}}, \text{ or}$$

$$x^3 y = 1, \; x^3 z = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Accordingly we have the system of fundamental equations:

$$\left.\begin{array}{l} H = y + kz + x, \text{ plane of momentum,} \\ E = y^2 + kz^2 - \frac{1}{x^2}, \text{ surface of energy,} \\ x^3 y = 1,\ x^3 z = 1, \text{ curve of rigidity.} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots (4).$$

These equations represent all possible interactions of the system, but in their present form are very difficult to interpret. The general problem to which they give rise seems to be insoluble, but we can solve and interpret them fully for one particular case which is in close accord with the conditions existing in nature; and it is possible to show by analogy that all other cases will be essentially similar to the one of which we shall treat.

By taking the case of two equal stars rotating in the same direction with equal angular velocities, or substituting (3) of (4) in (1) of (4), we reduce the plane of momentum to a particular line of that plane:

$$x^4 - Hx^3 + (1 + k) = x^4 - Hx^3 + 2 = 0, \text{ since } k = 1.$$

The equation of the energy surface passes into the form

$$E = \frac{(H - x)^2}{2} - \frac{1}{x^2}.$$

The curve of rigidity becomes

$$\eta = \frac{H - x}{\sqrt{2}}, \text{ where } \eta = \sqrt{y^2 + z^2}.$$

Every point in the plane of momentum represents one configuration of the system, *i. e.*, one distance apart, one velocity of axial rotation, one moment of momentum of orbital motion. This point therefore determines the dynamic condition of the system, and by the motion of this point we may discover the changes which are taking place in any case that may be imagined. As we have restricted the plane of momentum to one line, the guiding point representing the configuration of the system will simply glide back and forth along this line. In the same manner the surface of energy is now restricted to a curve formed by cutting that surface by a certain plane; the guiding point that would slide along the energy surface is thus restricted to one line of the surface given by the transformed equation. [The reader who may desire to examine this question exhaustively must be referred to my *Inaugural Dissertation*, *Die Entwickelung des Doppelsternsysteme*, Berlin, 1893, R. Friedländer & Sohn.]

As the tides raised in the stars are subjected to frictional resistance, energy is

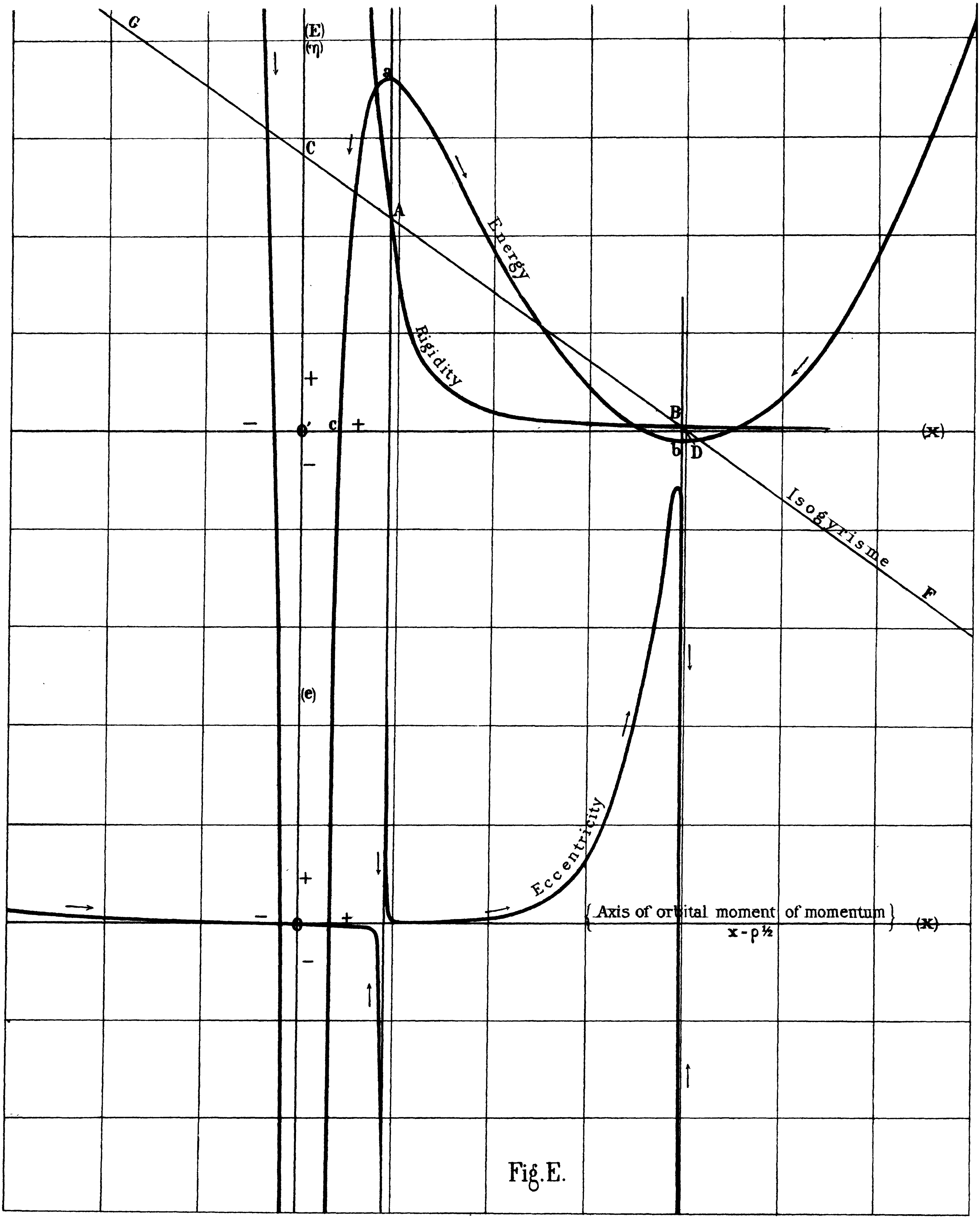


DIAGRAM FOR THE CURVES OF A SYSTEM OF EQUAL STARS, UNDER THE INFLUENCE OF TIDAL FRICTION.

Lower Curve illustrates increase of Eccentricity as the Stars separate.

thereby converted into heat, and lost by radiation into surrounding space; thus the total energy of the system must decrease with the time. Hence it follows that, however the system be started, the guiding point representing the configuration of the system must slide down a slope of the energy curve. In the accompanying illustration the curves are drawn for the value of $H = 4$.

If the guiding point is set at *a* it may move either of two ways: it may slide down the slope *ac*, in which case the stars fall together; or it may slide down the long slope *ab*, in which case the stars recede from each other under the influence of tidal friction. This latter case is the one of chief interest in respect to systems actually existing in space, and the several other ideal cases need not be discussed in this paper. The condition at *a* is dynamically unstable, and corresponds to that of the system at the instant when the stars are first separated. At this juncture they rotate as a rigid system, but as each is losing energy by radiation, the axial velocities will soon surpass the velocity of orbital motion, and then the tides will begin to lag, and the mutual reaction of the stars will drive them asunder. Thus the guiding point in general slides down the slope *ab*. This means that as the stars recede from each other, the period of revolution for a long time surpasses that of axial rotation, but that in time the two periods again become synchronous when the guiding point has reached the minimum of energy at *b*, where the bodies once more revolve as if rigidly connected.

The question now arises with respect to the changes of the eccentricity. The differential equation for the change of the eccentricity is shown to be

$$\frac{1}{e}\frac{de}{dx} = \frac{1}{2x}\left\{\frac{11\,x^3\,(H - x) - 36}{x^3\,(H - x) - 2}\right\},$$

which, on integration, is put into the form

$$e = \frac{Bx^{18}}{[x^4 - Ha^3 + 2]^{\frac{25}{8}}}\left\{\frac{(x \backsim a)^{\frac{H}{8a_1}}\, exp.\left[\frac{H\beta}{4\,(\alpha_1{}^2 + \beta^2)}\, arctan\,\frac{x - \alpha}{\beta}\right]}{(x \backsim b)^{\frac{H}{8a_1}}\left[(x - \alpha)^2 + \beta^2\right]^{\frac{Ha_1}{8\,(\alpha_1{}^2 + \beta^2)}}}\right\}^{\frac{25}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where B is an arbitrary constant; a, b, $\alpha \pm \beta\iota$, are the roots of the biquadratic equation. $x^4 - Hx^3 + 2 = 0$. Equation (5) is illustrated in the lower part of the preceding figure, the origin being shifted downward to O' to prevent confusion of too many curves in one diagram. Now as the guiding point on the energy curve slides down the slope *ab*, the eccentricity at first very slightly decreases, then increases slowly, finally much more rapidly, until a high maximum is reached, after which it again diminishes, owing to the libratory motion in the system. Thus it is clear that as the stars recede from each other, the orbit becomes highly eccentric, but will ultimately become circular when the system revolves as a rigid body. This last condition cannot come about while the stars are still contracting and shining by their own light, and hence all visible systems are characterized by highly eccentric orbits.

To leave no doubt that tidal friction is a sufficient cause to account for the elongation of the orbits of the double stars, I applied the theory to a special case, in which the masses, distances and velocities are known. Taking two spheroidal fluid masses each three times as large as the sun, expanded to fill the orbit of Jupiter, and set revolving in an orbit of 0.1 eccentricity at a mean distance of 30 astronomical units, I find that by tidal friction the major axis of the orbit will be increased to 48 astronomical units, while the eccentricity will rise to 0.57. In this problem the masses are set rotating at such a rate as will produce an oblateness of about $\frac{2}{5}$, so that the equilibrium is stable. Different conditions will produce different results, but it is easy to see by this numerical example that tidal friction is a sufficient cause to account for the observed elongation of the orbits of double stars.

Though it may be supposed that there could be little doubt of the generality of the law of the eccentricity which I inferred in 1888, yet the importance of this fundamental fact of the universe is so great that I did not feel satisfied till all the observations of double stars had been examined anew and this conclusion touching the eccentricity established upon the most unshakable foundation. At length I have been enabled to show by the most exhaustive investigation of stellar orbits ever attempted, that the most probable eccentricity is 0.48; while on the other hand extremely eccentric and extremely circular orbits are equally rare, and must be referred to some unusual circumstances. Thus of the 40 orbits now well-known, it turns out that none lie between the eccentricities 0.0 and 0.1; two between 0.1 and 0.2; four between 0.2 and 0.3; eight between 0.3 and 0.4; nine between 0.4 and 0.5; nine between 0.5 and 0.6; two between 0.6 and 0.7; four between 0.7 and 0.8; two between 0.8 and 0.9, and none between 0.9 and 1.0. It follows therefore that by whatever process the stars developed, their orbits assumed a form which is about a mean between the nearly circular orbits of the planets and the extremely elongated orbits of the periodic comets.

Now a double star can originate by but one of two processes: either such a system is the outgrowth of the breaking up of a common nebula, or it is made up of separate stars brought together in a manner analogous to that involved in the capture of a comet. That these systems are not the outgrowth of accidental approach of separate stars we may at once affirm; for if we suppose them to be so produced, there being no third disturbing body which acts like the sun in the capture of comets, the captured star would recede to a distance equal to that from whence it came. In that event we should observe stars moving in paths of very immense extent, and consequently revolving at the quickest in some hundreds of thousands of years. If the paths be elliptical, the major axes of these ellipses would be of the same order of magnitude as the distance which separates us from α Centauri; while if the paths be parabolic or hyperbolic, the two objects would pass and then separate forever. On the other hand we can conceive of nothing which could diminish the dimensions of a very long ellipse, unless it be something analogous to a resisting medium. Such a medium to be effective in reducing the size of the orbits would have to act for a great period of time, and besides would probably be visible in space as diffused nebulosity. No nebulosity is observed about revolving double stars, nor is there any evidence of a sensible resisting medium either among the stars or in our own solar system. We may therefore reject the idea that the dimensions of the orbits were originally very large, and have since been diminished. As the orbits are now of the size of those of our greater planets, and therefore comparatively small, it follows that the stellar systems have originated by some process other than by the union of separate stars.

As a nebula is a very rare and expanded mass, and is yet held in equilibrium by the pressure and attraction of its parts, it necessarily rotates very slowly; and hence when it divides into two parts under the acceleration of rotation due to secular condensation, the orbit pursued by the detached mass must be of small eccentricity. For even if the forces producing separation could be exerted suddenly to produce a violent rupture, the detached mass in pursuing its eccentric orbit would again come to periastron, where it would encounter resistance in its orbital motion, and the result of the grazing collision would be a diminution of the size of the orbit, and consequently an exaggeration of the resistance at the next periastron passage; in this way the system would very soon degenerate into one mass. On the other hand were the initial eccentricity small, the newly-divided masses would pass freely, and when the orbit eventually became highly eccentric the secular contraction in the size of the masses would prevent disturbance at periastron. Subsequent collision could not possibly occur, because the periastron distance would steadily though perhaps only slowly increase as the stars are pushed asunder and the orbit is rendered constantly more and more eccentric.

It follows therefore that in the beginning the orbits are only slightly eccentric, and that the eccentricity is developed gradually as the result of secular tidal friction working through immense ages. Accordingly in the elongation of the orbits now observed we see the trace of a cause which has been working for millions of years. The existence of this cause and its effects on stellar cosmogony could probably never be inferred except in the manner by which I approached the problem. On the one hand it appears that we have inferred the true cause of the expansion and elongation of the stellar orbits, while on the other the trace left by this cause has enabled us to detect the existence of unseen tides in every part of the heavens. In a fluid universe tides necessarily result from gravitation, and are as universal as this great law of nature. In my later researches I have therefore been much concerned to show from the discussion of reliable observations that gravitation is really universal* and consequently that the tides we have assumed actually exist in the bodies of the stars. It is thus made certain that the foundation upon which our cosmogonic speculation rests is as enduring as the Newtonian theory itself.

We now come to the second part of the problem: By what process did the stars separate? In college lectures I had heard the annular theory of Laplace expounded for the solar system, and yet I failed to see how this theory could account for the separation of equal or comparable masses, such as we observe among the stars. Realizing that the double stars are in fact made up of two bodies of comparable mass, I reached the conclusion while still at the Missouri University that there must exist some process by which a nebula divides into equal or comparable parts, in a manner analogous to that of fission among the protozoa. About November, 1889, very soon after I entered upon my studies at the University of Berlin, I found that Darwin had recently published an important mathematical paper on the figures of equilibrium of rotating masses of fluid, and had referred therein to the profound work of Poincaré published about a year before. When I beheld the figures of equilibrium which these mathematicians had computed, I recognized at once the cosmical process I had already assumed to exist; it was indeed a great satisfaction to see a demonstration that under gravitational contraction homogeneous incompressible fluid masses may divide into equal or comparable parts. The next question was: Are there nebulæ of this form in the actual universe? In searching over the paper of Sir John Herschel in the Philosophical TRANSACTIONS for 1833, I found some drawings of double nebulæ almost exactly like the figures mathematically determined by Darwin and Poincaré. It was no longer possible to doubt that the real process of double-star genesis had been discovered. Further investigation and reflection have confirmed this inference, and I believe we may now accept with entire confidence the result reached at Berlin in November, 1889.

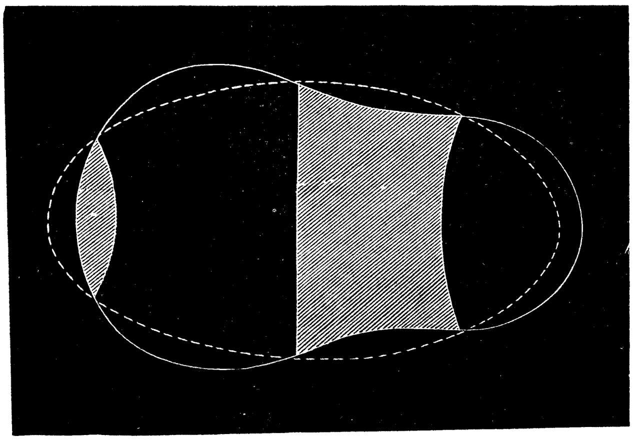
In the first investigation Poincaré begins with the Jacobian ellipsoid of three unequal axes, and imagines it shrinking in such a way as to remain homogeneous, and yet gain constantly in velocity of axial rotation. When the oblateness has become about $\frac{2}{5}$ he finds that the equilibrium in this form becomes unstable, and the mass tends to become a dumb-bell with unequal bulbs—an unsymmetrical pear-shaped figure which I have called the Apioid. As the contraction continues the whole evidently ruptures into two comparable masses, and the smaller will then revolve orbitally about the larger. If

* RESEARCHES ON THE EVOLUTION OF THE STELLAR SYSTEMS, Vol. I: *On the Universality of the Law of Gravitation and on the Orbits and General Characteristics of Binary Stars* (The Nichols Press, Lynn, Mass., 1896).

we suppose either mass to contract still further, it is evident that the rotation will begin to exceed the orbital motion; and the tides raised in either mass by the attraction of the other will lag, and tidal friction will henceforth play just the part we have already described.

Starting from a different point of view, Darwin was already at work on essentially the same problem when Poincaré's paper appeared, and he held his results back for nearly a year longer, hoping to make application of the principle Poincaré had announced. In this second method of treatment two masses of homogeneous fluid were brought so close together that the tidal distortions of their figures caused them to coalesce into one mass; set in motion as a rigid system, the problem was to find the resulting figure of equilibrium. It turned out to be a dumb-bell with equal or unequal bulbs according to the relations of the primitive masses. Thus we see it proved from two

Fig. 1.



The Apoid of Poincaré, showing how a rotating mass of fluid separates into two unequal parts.

independent points of view that a division such as I assumed in 1888 can theoretically take place; and among actual nebulæ of space such division seems to be a general law. During the years of 1896 and 1897, I have examined a number of such objects in the southern hemisphere, and find them substantially as drawn by Herschel many years ago. Burnham and Barnard had previously assured me that the interpretation of the figures of double nebulæ based on the drawings of Herschel was in accord with the phenomena of nature, but the studies more recently made with the great Lowell telescope supplements their large experience in a very happy manner, and may be said to remove the last doubt that could attach to the division of nebulæ by the process of fission.

Before concluding these remarks it ought to be pointed out that in space we have to deal with masses which are not homogeneous, nor are the nebulæ by any means incompressible; yet many considerations lead us to believe that in most cases the density of a nebula is not very heterogeneous, and hence in general the foregoing conclusions would not be greatly modified. In this reasoning I have assumed nothing but that the nebulæ are figures of equilibrium under the action of gravitation. That these masses are fluid is certain, for the bright lines of their spectra indicate that they are self-luminous gas; on the other hand the same force which controls the motions of the stars must operate among the particles of the nebulæ, and thus determine the figures of the masses in accordance with the laws of mechanics.

As the conditions here assumed certainly exist in the heavens, we need only add that when the masses separate they are probably revolving as a rigid system. When they contract under the influence of gravitation, they must by a well-known mechanical law gain in velocity of axial rotation, and tidal friction then begins expanding and elongating the orbits; in the course of some millions of years we have a double star like α Centauri or 70 Ophiuchi.

The stellar cosmogony here suggested may be regarded as a very general theory. Our solar system is so remarkable that it is uncertain whether a theory which explains the formation of double stars could assign also the cosmogonic processes which have given birth to the planets and satellites. The masses of the planets are very small compared to that of the sun, and the masses of the satellites are equally insignificant compared to those of the planets about which they revolve. Moreover the orbits are very circular, and these various circumstances make our system absolutely unique in the known creation. Yet so far as our researches on the double stars may illuminate the problem of planetary cosmogony, they indicate that the separation took place in the form of lumpy or globular masses—not in rings or broad zones of vapor such as Laplace supposed.

From the survey thus hastily made of a very large subject, it appears that we have taken a step in the generalization of the theory of tides and of tidal friction, and have indicated the probable mode of formation of the stellar systems. Little or nothing is known of the development or even of the mechanism of star clusters; the problem of explaining the more complicated systems must ultimately occupy the attention of astronomers if we are ever to trace the development of the visible universe. As a step in the direction of accounting for the origin of multiple systems, it may be said that observations on triple and quadruple stars have shown that they, too, developed by repetition of the fission process. One or both components of a binary have again subdivided, just as I inferred was the case when still at the Missouri State University in 1888. While the views here expressed are the results at which I have arrived after a partial investigation of the theory of tides and of the figures of equilibrium of rotating masses of fluid and a comparison of these theories with the phenomena observed in the heavens, I reserve the right to modify any opinion or conclusion which future research may show

Drawings of double nebulæ according to Sir John Herschel.

to be unsound or incomplete. That tidal oscillations which were first noticed by the navigators of our seas are at length seen to be but special phenomena of a general law operating throughout the universe is alike honorable and gratifying to the human mind. It is equally inspiring to recall that by the known laws of these phenomena we are enabled to trace existing systems through immeasurable time, and thus disclose cosmical history which mortal eye could never witness. In our time it is no longer sufficient to maintain the traditions of the past, to trace the planets, satellites and comets through centuries, and explain observed anomalies in their figures, attractions and orbital motions by the law of gravitation. We must essay to discover the cosmical processes by which the existing order of things has come about. Though it seems probable that a fair beginning on this problem has already been made, a much greater work remains to be done during this and the coming century.

What is needed is a more thorough exploration of the face of the heavens, by astronomers who are familiar with the laws of mechanics; and a far-reaching investigation of the general theory of tides in viscous liquid and gaseous masses such as the stars and nebulæ of remote space. Even if the full extent of the hopes here expressed can be realized only after the lapse of several centuries, I venture to believe that the achievement will not be unworthy of the past history of Physical Astronomy.